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MAGNETICALLY DRIVEN FLYERS - THEORY AND APPLICATION(U)

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ABSTRACT

The term magnetically driven flyers generally refers to conductive, non-magnetic metals which are driven by the force created when a current flows in the presence of a magnetic field (motor principle). In practical usage, these flyer materials are of the order of 10 mils thick. The flyer material is spaced away from the specimen a distance such that the current flow through the flyer material has essentially just reached zero (i.e., the flyer is free of the forcing function). Any excessive spacing is avoided to minimize flyer unstability such that flyer simultaneity can be closely controlled. In the case of metallic test specimens, high voltage arcing problems can also be a consideration in determining the proper spacing.

The electrical energy source for a magnetic flyer facility is typically a high energy capacitor bank. The circuit resistance is minimized such that an underdamped L-C circuit is formed. The forcing function driving the flyer is then a series of pulses corresponding to the sinusoidally varying current. By crowbarring the load, the forcing function may be converted to a single smooth pulse although this is generally difficult to accomplish. The ratio of load inductance change to residual inductance should be as high as possible for optimum efficiency.

By utilizing proper design techniques, the magnetic flyers are applicable for one, two or three dimensional geometries. Load variations, such as a cosine load distribution, are also possible.

The theoretical limitation on the maximum flyer velocity is imposed by I^2R heating of the flyer and/or the "magnetic saw" effect (i.e., localized cracking in the material due to the pressure of the high magnetic fields). Typical velocities obtained to date are of the order of 1 mm/μsec. Since the effect of I^2R heating is influenced by material parameters (resistivity, heat capacity, etc.), the theoretical maximum flyer velocity is a function of the material being used for the flyer.

Ignoring instrumentation and manpower costs, the economics of magnetic flyer plate testing is determined by the complexity of the flyer plate fixtures. In general, system maintenance costs are low and electrical energy is typically obtained for approximately one cent/megajoule. In the case of one dimensional testing (typically 1 to ~ 6 in^2), the permanent and reusable portion of the fixture is available for something on the order of $100 and per shot costs should total less than $10 for flyers. At the other extreme, machining and material costs for complex loading, three dimensional tests could easily cost in the thousands of dollars range.

The magnetic flyer testing method is advantageous from the viewpoint of predictability and repeatability. Simultaneity is reported to be in the tens of shakes region for a simple geometry and it is reasonable to expect this number to decrease with increasing technology.

INTRODUCTION

This report presents a brief description of the theory and application of magnetically driven flyer plates. Included in the application discussion is a comparison of other similar test techniques which are also being used to generate short duration impulsive loads.

I. MAGNETICALLY DRIVEN FLYER PLATE THEORY

Magnetic Pressure

In the case of an infinite parallel plate geometry, the magnetic field generated by currents flowing in opposite directions is contained between the plates as shown in Figure 1.

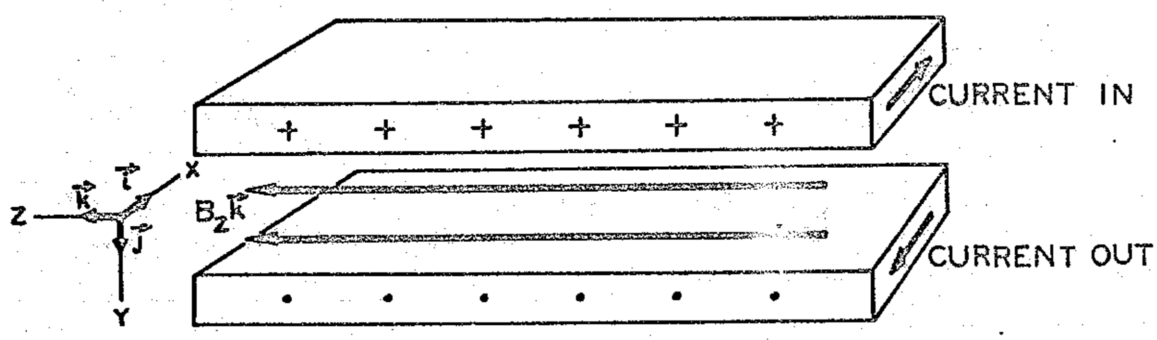


Figure 1

Current - Magnetic Field Relationship With Infinite Plate Geometry

Combining the basic force equation

$$\vec{F} = \vec{i} \times \vec{B} \quad (1)$$

with Maxwells equation governing the relationship between current and magnetic field $\vec{H}$

$$\vec{i} = \vec{\nabla} \times \vec{H} = \frac{1}{\mu} \vec{\nabla} \times \vec{B} \quad (2)$$

we have the following expression for the force per unit volume

$$\vec{F} = \frac{-1}{2\mu} \frac{\partial B_z^2}{\partial y} \vec{J} \quad (3)$$

where

$\vec{F}$ = force per unit volume

$\vec{i}$ = current density

$\vec{B}$ = flux density

$\vec{H}$ = magnetomotive force

μ = permeability

Then, integrating through the thickness of the conductor, we obtain the force per unit area (pressure)

$$\frac{d\vec{F}}{dA} = \vec{P} = + \frac{1}{2\mu} B_z^2 \vec{j} \qquad (4)$$

Therefore, the pressure exerted on either conductor is proportional to the square of the field strength and tends to force the conductors apart.

Utilizing the basic expression for the energy density (w) contained in a magnetic field, we see that

$$w = \frac{1}{2} BH = \frac{1}{2\mu} B^2 \qquad (5)$$

Therefore, we have the interesting relationship that the "magnetic pressure" is numerically equal to the energy density contained in the magnetic field. This situation is exactly analogous to the case of a contained gas. This analogy is extremely useful in envisioning the forces created in a current carrying system.

It is also useful to obtain the forcing function in terms of the circuit parameters as follows.

The energy stored in the magnetic field of an inductor is given by

$$W = \frac{1}{2} LI^2 \tag{6}$$

where

W = energy
L = inductance
I = current

Then, the force (f) created (neglecting the vector notation for simplicity) is

$$f = \frac{\partial W}{\partial y} = \frac{\partial}{\partial y}\left(\frac{1}{2} LI^2\right) = \frac{1}{2} I^2 \frac{\partial L}{\partial y}\bigg|_{I = \text{constant}} \tag{7}$$

The expression for the inductance of a closely spaced parallel plate geometry is

$$L = \mu y \frac{\ell}{b} \tag{8}$$

where

y = spacing between plates
ℓ = length of the plates
b = width of the plates

Then, in the case where current is constant, the total force is given by

$$f = \frac{1}{2} I^2 \frac{\partial L}{\partial y} = \frac{1}{2} I^2 \mu \frac{\ell}{b} \tag{9}$$

or in terms of pressure

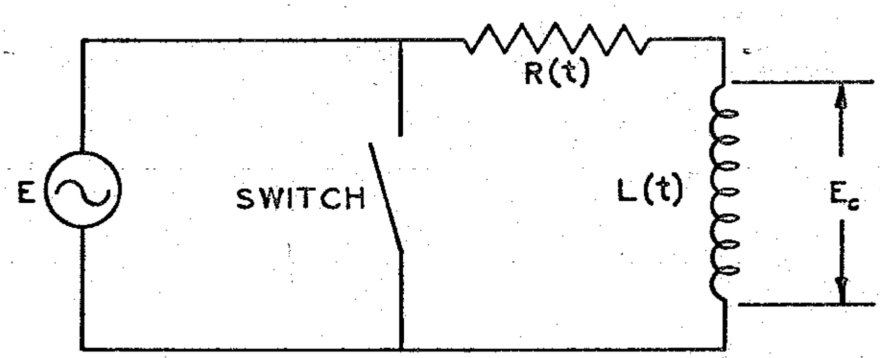
$$P = \frac{\mu}{2b^2} I^2 \tag{10}$$

Equation (10) gives some insight into the behavior of magnetic pressure since the only dimension involved is the width (b) of the magnetically driven flyer plate provided that the plates are closely spaced and that a given current flow exists.

Electric Efficiency

Efficiency may be studied either from energetics or circuit concepts[1]. For the purpose of this discussion, we shall use circuit analysis.

Consider the following circuit:



1. Edited by Kolm, H., Lax, B., Bitter, F. and Mills, R., High Magnetic Fields, MIT Press and John Wiley & Sons, 1962, Chapter 22, "Pulsed Magnets" by H. P. Furth.

where a current is flowing through R(t) and L(t) which represent the time varying resistance and inductance of the magnetic flyer.

For a single turn coil, we have

$$E_c = \frac{d\Phi}{dt} \tag{11}$$

where Φ is the total flux of the coil and

$$L = \frac{\Phi}{I} \tag{12}$$

The energy stored in the resistance (heating) is

$$W_R = \int_o^t I^2 R \, dt \tag{13}$$

The energy input to the time varying inductance is given by

$$\int_o^t E_c I \, dt = \int_o^t I \frac{d(LI)}{dt} dt$$

$$= \int_o^t I^2 \frac{dL}{dt} dt + \frac{1}{2} \int_o^t L \frac{dI^2}{dt} dt \tag{14}$$

Integrating the second term on the right by parts and combining the integrals, we have

$$\int_0^t E_c I \, dt = \frac{1}{2} LI^2 \Big|_0^{t_1} + \frac{1}{2} \int_{L_o}^{L_o + \Delta L} I^2 \, dL \tag{15}$$

$$= W_s + W_{M.W.} \tag{16}$$

where

W_s = energy stored in the magnetic field

$W_{M.W.}$ = kinetic energy of the flyer

W_R = energy dissipated in the form of heat

If we ignore, for the moment, the W_R term and re-examine Equation (14), we can develop an intuitive feeling for the efficiency of the system.

Assuming the inductance changes from L_o to $L_o + \Delta L$ in time t and that the current I is independent of inductance, we have

$$W_s = \frac{1}{2}\left(L_o + \Delta L\right) I^2 \tag{17}$$

and

$$W_{M.W.} = \frac{1}{2} I^2 \Delta L \tag{18}$$

Then, efficiency

$$\epsilon_1 = \frac{W_s}{W_s + W_{M.W.}} < \frac{\Delta L}{2\Delta L + L_o} \qquad (19)$$

Then, $\epsilon_1 \rightarrow 50\%$ as $L_o \rightarrow 0$. (In an actual circuit, L_o should be considered to be the total inductance of the circuit.)

If we now conduct the experiment in such a way that the current is brought to a peak value before the flyer has moved appreciably and then short circuit the inductor (close the switch), the current will continue to flow in the inductor and Φ then becomes a constant. [Φ is a constant since the flux lines are trapped inside of the closed circuit as is shown by setting $E_c = 0$ in Equation (11)]. Using Equation (12), Equation (15) becomes

$$0 = \frac{1}{2} \frac{\Phi^2}{L} \Bigg|_{t_1}^{t_2} + \frac{\Phi^2}{2} \int_{L_o}^{L_o + \Delta L} \frac{dL}{L^2} \qquad (20)$$

$$= \frac{\Phi^2}{2} \left[\frac{1}{L_o + \Delta L} - \frac{1}{L_o}\right] - \frac{\Phi^2}{2} \left[\frac{1}{L_o + \Delta L} - \frac{1}{L_o}\right] \qquad (21)$$

$$\therefore \frac{\Phi^2}{2L_o} = \frac{1}{2} L_o I_o^2 = \frac{\Phi^2}{2} \left[\frac{1}{L_o + \Delta L} + \frac{\Delta L}{L_o(L_o + \Delta L)}\right] \qquad (22)$$

$$= W_s + W_{M.W.}$$

Then, the efficiency becomes

$$\varepsilon_2 = \frac{W_{M.W.}}{W_{M.W.} + W_S} < \frac{\Delta L}{L_O + \Delta L} \qquad (23)$$

Then, $\varepsilon_2 \rightarrow 100\%$ as $L_O \rightarrow 0$.

As mentioned previously, the efficiency expressions ε_1 and ε_2 assume no I^2R losses in the circuit.

The efficiency expressions ε_1 and ε_2 demonstrate the value of a low inductance system for efficient operation of a magnetically driven flyer facility.

In practice, short circuiting the load (commonly called crowbarring) to increase efficiency is difficult to accomplish with the high current flows (10^5 - 10^6 amperes) normally associated with magnetically driven flyers. The difficulty arises due to the fact that any switching network inherently has impedance. For instance, if we assume a 1 nanohenry switch (which is very low for a practical system) and assume that we want to switch a current of 10^6 amperes in 10 nanoseconds, then the voltage developed is

$$E = L \frac{dI}{dt} \simeq \frac{10^{-9}\ 10^{6}}{10^{-8}} = 10^5 \text{ volts} \qquad (24)$$

This high voltage is especially discouraging when it is realized that zero voltage exists across the load at the time the current is maximum (assuming a pure L-C discharge circuit). On the more optimistic side, crowbarring has been successfully employed on capacitor banks with relatively slow discharge frequencies and the possibility does exist that crowbarring may be employed on magnetically driven flyer plate facilities by making compromises in the switching time and the current amplitude at the time of switching.

With present day technology, efficiences in the 1% to 30% range may be accomplished.

<u>Flyer Heating</u>

Since the principal limitation which exists in obtaining high velocity magnetically driven flyers is the heating of the flyer due to I^2R losses, the ratio of kinetic energy/potential energy of the flyer is of primary importance. An estimate of the velocity limitation due to heating of the flyer may be obtained by the following simplified analysis.

From kinematics:

$$v = \frac{A}{m} \int_0^t P \, dt = \frac{A\mu}{2mb^2} \int_0^t I^2 dt \tag{25}$$

where

A = area

m = mass

b = width

From low temperature thermodynamics for a solid

$$\int_0^t I^2R\,dt = W_R = \frac{\ell}{ba}\int_0^t I^2 r_o(1 + \alpha\tau)\,dt$$

$$= m\,\Delta H \simeq (H_o + \beta\tau) \qquad (26)$$

where

α = temp coefficient of resistivity

r_o = initial resistivity

ℓ = flyer length

τ = temperature

ΔH = enthalpy change/unit mass

a = flyer thickness

$$\therefore\ m\,\frac{d\,\Delta H}{d\tau}\,\frac{d\tau}{dt} = \frac{\ell}{ba}\,I^2 r_o(1 + \alpha\tau) = m\beta\,\frac{d\tau}{dt} \qquad (27)$$

$$\therefore\ \frac{m\beta}{r_o\ell}\int_{\tau_1}^{\tau_2}\frac{d\tau}{(1 + \alpha\tau)} = \int_0^t I^2\,dt \qquad (28)$$

Combining Equations (25) and (28) we have

$$v = \frac{\mu\beta a}{2r_o\alpha}\Big[\ell n(1 + \alpha\tau)\Big]_{\tau_1}^{\tau_2} \qquad (29)$$

The significance of Equation (29) is that the velocity is expressed in terms of temperature and flyer thickness but that it is independent of the current waveform. It should be noted that this expression was developed on the basis of uniform heating of the foil (i.e., skin depth and diffusion effects are neglected). Inclusion of the skin depth and diffusion effects would cause a non-linear heat distribution with the inner surface being the hottest. Equation (29) may be used to predict velocities of approximately 1.5 mm/μsec with .010" thick flyers of either copper or aluminum prior to start of melt.

II. MAGNETICALLY DRIVEN FLYER PLATE APPLICATION

Magnetically driven flyers are applicable for the creation of relatively short duration impulsive loads on one, two or three dimensional test objects. This test technique offers the advantages of (1) repeatable test conditions, (2) predictability and (3) controlled and well defined pressure pulses — provided the equation of state and thermodynamic conditions of the flyer material are known. These advantages stem from the well defined behavior of linear electrical circuits. Therefore, one of the chief problems associated with magnetically driven flyer plate testing is technological, i.e., to maintain a high energy system capable of discharging in a consistent manner.

One Dimensional Testing

In the case of one dimensional equation of state and material damage studies, Sandia-Livermore has demonstrated the usefulness of the magnetically driven flyer testing technique. Measured test parameters and material responses are showing repeatable results. SCLL is quoting a non-planarity of 0.2° in the one dimensional flyers. Therefore, the non-planarity of one dimensional magnetically driven flyers falls between the excellent planarity obtainable in gas guns and the relatively poor planarity of exploding foil driven flyer plates. Since the "mag flyers" are limited to conductive flyer plate materials, it forms an excellent counterpart to the exploding foil driven flyers which are non-conductive. Although exploding foil driven flyer plates are capable of higher velocities than "mag" flyers, the normal difference in impedance between organic non-conductive flyers and metallic flyer materials tend to equate the two test techniques in terms of pressure generation in a given target material. Although neither of these two types of flyer plate facilities can, with present technology, rival the gas gun for precisely controlled test conditions, they are, in general, faster and cheaper to operate.

A comparative summary of these three common test techniques is shown in Table I. With present day technology, all three of these techniques have a useful and complementary position in a well rounded one dimensional materials testing program. The gas gun is a refined instrument which is rather slow and costly to operate but is capable of producing accurate and repeatable results. The exploding foil facility has proven itself capable of producing reliable damage threshold results with fast and inexpensive tests, although several tests are generally required due to shot-to-shot non-repeatability. The magnetically driven flyer plate type of facility offers fast and economical tests together with flexibility and sophistication approaching that of the gas gun. The "mag flyer" concept is also attractive in

TABLE I

COMPARATIVE SUMMARY OF NON-EXPLOSIVE
PLATE TESTING TECHNIQUES FOR ONE DIMENSIONAL MATERIALS TESTING*

Type	Maximum Impact Velocity	Flyer Type	Pressure Pulse Width	Planarity	Relative Operating Costs	Ease of Instrumentation
Conventional Gas Gun	~ 1 mm/μsec	Any	~ 0.05 → 10 μsec	Good	High	Good
Exploding Foil Driven Flyers	~ 4 mm/μsec	Dielectric	~ .04 → .08 μsec	Poor	Low	Poor
Magnetically Driven Flyers	~ 1 mm/μsec	Conductive	~ .05 → .2 μsec	Fair	Low → Medium	Fair

Note: Due to the low mechanical impedance of the normal exploding foil driven flyers, the peak pressures available from these three facilities tend to be the same.

*The intention of this table is to present a generalized comparison. Exceptions and qualifications do exist but, in the interest of simplicity, are not included in the table.

that the same energy source may be used for exploding foil driven flyers; hence, one energy source can provide the means to test with both conductive and dielectric flyer materials with a minimum expenditure.

Two and Three Dimensional Testing

Two and three dimensional magnetically driven flyer plate testing is now being conducted at EG&G, under the sponsorship of Sandia-Livermore. These experiments are being conducted on energy sources as large as 100 KJ.

No theoretical limitation is in sight as to the maximum size of a two or three dimensional magnetically driven flyer plate. The thickness of the flyers is determined by the desired pulse shape during impact, temperature of the flyer and stability considerations.

The geometry of two and three dimensional magnetically driven flyers is similar to the one dimensional fixture except that the flyer is curved to conform to the shape of the test specimen. Since the flyer plate must now move in at least two dimensions to conform to the test specimen, hoop stresses exist in the flyer prior to impact. Thus, the experiment designer must be careful to insure that impact occurs before the internal stresses in the flyer cause flyer instability. Similarly, there is an implication that the analysis of test results may need to consider the presence of stresses in the flyer.

Spatial load variations, e.g., cosine loadings, are possible with magnetically driven flyers by appropriately shaping the current distribution in the flyer. In general, lower efficiency is the penalty incurred to obtain spatially varying loads.

An accurate comparison of magnetically driven flyer plate testing with the various thin sheet explosive loads is somewhat difficult due to the lack of definitive, and well diagnosed, data for the various test methods. The best estimate of relative characteristics of the different methods is shown in Table II.

In the case of the two dimensional type of testing, the magnetic flyer facility offers a desirable combination of impulse and pulse duration. In its present day state, the simultaneity appears adequate for most types of structural testing and is expected to improve with additional development efforts.

It is difficult to compare operating costs for the various types of two and three dimensional types of testing. The initial cost of a capacitor bank assembly is rather high (20 cents to $1/joule), although the facility can generally be located within existing laboratories. Capacitor bank operation is considered to be safer than handling explosives and they do not present a problem in being in accordance with city and county ordnances. Explosives sites generally require

TABLE II

COMPARISON OF STRUCTURAL IMPULSIVE LOADING METHODS*

Type	Impulse Range	Pulse Duration	Simultaneity	Ease of Instrumentation	Relative Operating Costs
**Exploding Foil Initiated Secondary Explosives	$\sim > 10^4$ taps	$\sim .5 \rightarrow 5$ μsec**	$\sim < 1$ μsec	Poor	Low → Medium
**Light Initiated Primary Explosives	$\sim 10^2 \rightarrow 10^4$ taps	$\sim 1 \rightarrow 5$ μsec	$\sim < 2$ μsec	Fair	Low → Medium
**Running Load Secondary Explosives	$\sim > 10^4$ taps	> 1 μsec	(Detonation velocity of ~ 7 mm/μsec)	Good	Low → Medium
Magnetically Driven Flyer Plates	$\sim 10^2 \rightarrow 3 \times 10^4$	$\sim .05 \rightarrow .02$ μsec	$\sim < 1$ μsec	Fair	Low → High

*The intention of this table is to present a generalized comparison. Exceptions and qualifications do exist but, in the interest of simplicity, are not included in the table.

**Assumes solid sheets (no stripping), no attenuators and no containment.

a semi-remote type of operation which may require special building and site preparation. Magnetically driven flyers present a wide range of day to day costs depending on the experiment. This range probably goes from the cheapest to the most expensive when comparing with the other types of testing.

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